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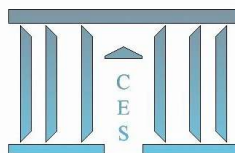
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## Abstract

In the case of a large nuclear set (like the French set), nuclear production needs to be flexible to adjust to the predicted evolutions of the energy demand. Consequently, the dominant position of nuclear in the national energy mix makes it responsible for the overall equilibrium of the electricity system which is directly intertwined with social welfare. In a previous work, we looked at producers own profits (short-term, inter-temporal) considering the equality between supply and demand. Here, we proceed with a full optimization of the social welfare in an identical framework. Theoretically, the optimal production behaviour that maximizes social welfare is characterized by a constant thermal production and a totally flexible nuclear production given that the nuclear capacity is sufficient. Numerically, the significant amount of nuclear capacities compared with thermal capacities in the French electricity market leads to the same “paradoxical” production behaviour. Therefore, we conclude that social optimum is ensured within our model by investing sufficiently in nuclear capacity. The optimal production scheduling determined by the social welfare maximization problem and the optimal inter-temporal production problem are totally opposite.

**Key words:** Electric power, nuclear power plant, flexibility, nuclear fuel stock, thermal generation, social welfare, total cost minimization.

**JEL code numbers:** C61, C63, D24, D41, L11.

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# 1 Introduction

In France, nuclear power plants do not only operate at baseload but they also need to be flexible<sup>2</sup> to follow a part of the variable demand because of the very significant size of the nuclear set (IEA (2008), Pouret and Nuttall (2007), Bruynooghe et al. (2010)). This permits to ensure the overall equilibrium between supply and demand and therefore to avoid a potential “blackout”. The monitoring report realized by the French energy regulator (CRE) in 2007 gives an illustration of the operation of the French nuclear set (Regulatory Commission of Energy (2007)). It illustrates the way that the nuclear generation set is managed in France, focusing on the load-following ability which characterizes it. In the Ph.D. thesis of Lykidi (2014), we determined the optimal production behaviour when producers maximize their profits (short-term, inter-temporel) in a deregulated electricity market dominated by the nuclear energy. Here, the optimization no longer considers only the benefits of the generators, it now takes into account the “benefits” for the whole society: social welfare. The nuclear operators being the main producers of electricity in their national electricity market may have to consider constraints inherent in the public interest when they optimize the management of their nuclear fleet even in a competitive market. Such a constraint already covers the equality between supply and demand taking into account the threat of a “blackout”. Indeed, this constraint is already considered in the determination of the optimal management of the nuclear production. It covers both the short-term and the inter-temporal optimization of the nuclear production. However, the production decisions of a very large nuclear set have considerable consequences for the whole national electricity system and hence for the welfare of the society. This may lead towards a complete optimization of the social welfare instead of the producers own profits. The question of the management of the nuclear set in order to maximize social welfare could be also motivated by the fact that the social acceptability<sup>3</sup> of nuclear is never acquired definitively and until now the economic arguments has not given a decisive answer whether or not nuclear has to participate in the energy mix of a country. Therefore, through this new problem, we determine the optimal production levels that maximize social welfare given that the thermal capacity (e.g. coal, gas, etc.) as well as the nuclear capacity are exogenous.

In view of the periodical shutdowns of nuclear reactors for reloading their fuel, we introduce, in the medium-term, the feature of the nuclear fuel “reservoir” - partly similar to an hydro-reservoir. We aim to determine the management of the nuclear fuel reservoir in order to maximize social welfare during the time horizon of our model which consists of a number of campaigns. Each campaign corresponds to the period of production between two successive moments of reloading. Its length is given by the maximum number of days during which a nuclear unit produces until exhaustion of its fuel of reloading. It generally takes between 12 and 18 months (Source: EDF, CEA (2008)). During this period, nuclear producers have to set their seasonal variation of reservoir’s nuclear fuel to satisfy the seasonal demand and maximize their profits. From the modelling of the nuclear fuel reservoir result constraints intrinsic to the inter-temporal management of the nuclear fuel stock during several campaigns. Moreover, we take into account production constraints imposed by the flexible<sup>4</sup> operation of nuclear reactors.

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<sup>2</sup>The new reactor EPR, which is an evolution of the pressurized water reactor (PWR), is an example of a III+ generation nuclear reactor which is designed to accommodate load-following operation (AREVA (2005), Goldberg and Rosner (2012)).

<sup>3</sup>Germany is a typical example of a country which shuts down all its nuclear plants after the Fukushima disaster because nuclear was no more acceptable to society.

<sup>4</sup>A nuclear unit can vary its capacity level between the nominal capacity and the technical minimum. In the case of an EPR, load follow enables planned variations in energy demand to be followed and can be activated between 25% of nominal capacity (technical minimum) and 100% of nominal capacity (technical maximum)

We also include in the set of production constraints, the constraints issued from the available thermal generation capacity and the equality between supply and demand each month. The maximization of social welfare could give some insights with respect to an alternative behaviour of the nuclear production (optimal short-term production behaviour, optimal inter-temporal production behaviour).

Section 2 provides a brief description of our modelling and of the assumptions made within our model. A more detailed description can be found in the second chapter of the Ph.D. thesis of Lykidi (2014) where we build our model and we look at the optimal short-term management of flexible nuclear plants in a competitive electricity market as a case of competition with reservoir. Therefore, the demand for electricity and the time horizon of the model and of the campaign are themselves modelled in the same way. We also bring to mind the modelling of the generating units and of the production costs is presented in the general case of  $N \geq 2$  producers. In section 3, we proceed with the maximization of social welfare. First, we define the set of feasible solutions and of its interior. Then, we look at the social welfare maximization problem under the set of feasible solutions. In view of our hypothesis of perfectly inelastic demand, we show that its resolution is not possible. This leads us to the resolution of an equivalent optimization problem which consists of the minimization of the total production cost. Theoretically, in the case that the production constraints are not saturated, we come up with a novel property that entirely characterizes the optimal solutions of the social welfare maximization problem. Numerically, we feed our model with some data in order to resolve the social welfare maximization problem by using Scilab<sup>5</sup> (Section 4). Simulation results of the social welfare maximization problem are analyzed and then, they are compared with the theoretical results and the results of the optimal inter-temporal production problem. In section 5, we conclude.

## 2 Model: Perfect competitive case

In this section, we present shortly our deterministic, dynamic model and the assumptions made within it. We consider a perfect competitive market where each producer disposes an amount of nuclear and thermal capacity. The price in the market is calculated each month according to the merit order price rule<sup>6</sup>. We aim to specify the actions that the social planner will take to manage a large and flexible nuclear set in order to maximize the welfare of the society. Nuclear plants operate at baseload and at semi-base load and thus, they respond to a part of the variable demand. In particular, we take into account the medium-term horizon in which nuclear follows the variations of seasonal demand. In the determination of the optimal production behaviour that maximizes social welfare we look at the management of the nuclear fuel reservoir during several campaigns of production. A number of operational constraints regarding the inter-temporal management of the nuclear fuel stock (nuclear fuel storage constraints), the flexible operation of nuclear units and the generation capacity (minimum/maximum pro-

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(NEA/AEN (2011)).

<sup>5</sup>Scilab is an open source, cross-platform numerical computational package and a high-level, numerically oriented programming language. It can be used for numerical optimization, and modelling, simulation of dynamical systems, statistical analysis etc.

<sup>6</sup>The merit order is a way of ranking the available technologies of electricity generation in the same order as their marginal costs of production. This ranking results in a combination of different generation technologies to reach the level of demand at a minimum cost. The price in the market is therefore determined by the marginal cost of the “last technology” used to equilibrate supply and demand (perfect competitive case). This technology is also called marginal technology.

duction constraints) as well as the continuous and never ending equality between supply and demand (supply-demand equilibrium constraints) are considered in the maximization of the social welfare.

For simplicity reasons and in the absence of access to detailed data the electricity importations/exportations are not taken into account within our model. Moreover, our work centers only on the nuclear fuel storage and the optimal management of the nuclear fuel reservoir without considering the production coming from hydro units with possibility of storage (peaking<sup>7</sup> power plants) because of the additional capacity and storage constraints which would increase the complexity of the model. There exists an extensive literature that studies the optimal management of hydro-reservoirs in mixed hydro-thermal competitive markets and where one can see several modellings of the optimal production problem and notice the increased level of difficulty from a theoretical and numerical point of view (Arellano (2004), Bushnell (2003)). A more detailed justification of the overall assumptions of our model as well as mathematical proofs of propositions which appear in this section can be found in the second chapter of the Ph.D. thesis (Lykidi (2014)).

## 2.1 Modelling the generating units

We study a competitive electricity market with  $N \geq 2$  producers who manage both nuclear and thermal generating units. A producer  $n = 1, \dots, N$  can operate with all types of nuclear generating units. Moreover, each producer disposes of a certain amount of thermal capacity.

### 2.1.1 Concept of type

Among the nuclear generating units, we distinguish several essential intrinsic characteristics:

- available nuclear capacity,
- minimum capacity when in use,
- month of their fuel reloading.

In our model, the minimum capacity is proportional to the available capacity, and this proportion is the same for all “physical” nuclear reactors. Therefore, for each “physical” nuclear reactor, we will focus on the month of fuel reloading, which permits us to define twelve “types” of nuclear units. Each type indexed by  $j = 1, \dots, 12$  corresponds to a different month of reloading of the nuclear unit. Then, a unit which belongs to the type of unit  $j = 1$  (respectively  $j = 2, \dots, j = 12$ ) shuts down in the month of January (respectively February,  $\dots$ , December).

A nuclear plant<sup>8</sup> may contain several “physical” nuclear reactors, which (for operational reasons) do not reload on the same month. The characteristic “type” for the nuclear case is not related to the plant but to the reactor. Each producer  $n = 1, \dots, N$  owns a precise number of “physical” nuclear reactors that are grouped according to the month of reloading (independently of the locations) in order to constitute units. Therefore, it can hold a certain level of capacity from each type of nuclear unit.

The modelling regarding the thermal units is the same except that the minimum capacity is equal to zero and that there is no month of reloading. There is a unique type of thermal units.

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<sup>7</sup>Peaking power plants are power plants that generally run only when there is a high demand, known as peak demand, for electricity.

<sup>8</sup>A nuclear power plant is a thermal power station in which the heat source arises from nuclear reactions. A nuclear unit is the set that consists of two parts: the reactor which produces heat to boil water and make steam and the electricity generation system in which one associates: the turbine and the generator. The steam drives the turbine which turns the shaft of the generator to produce electricity (Source: SFEN).

## 2.2 Modelling the production costs

We recollect the modelling of the production costs. The cost functions of both nuclear and thermal production are common to all producers. The nuclear cost function is made of a fixed part determined by the cost of investment, the fixed cost of exploitation and taxes and a variable part which corresponds to the variable cost of exploitation and fuel cost. We assume that the cost function of the nuclear production is affine and defined as

$$C_{n,j}^{nuc}(q_{njt}^{nuc}) = a_{nuc}^{n,j} + b_{nuc} q_{njt}^{nuc}.$$

The thermal cost function is also made of a fixed part which corresponds to the cost of investment, the fixed cost of exploitation and taxes and a variable part covering the variable cost of exploitation, the fuel cost, the cost of CO<sub>2</sub> as well as the taxes on the gas fuel. We assume that the thermal production has a quadratic cost function  $C_n^{th}(\cdot)$  which is the following:

$$C_n^{th}(q_{nt}^{th}) = a_{th}^n + b_{th} q_{nt}^{th} + c_{th}^n q_{nt}^{th^2}.$$

The nuclear and thermal cost functions are monotone increasing and convex functions of  $q_{njt}^{nuc}$  and  $q_{nt}^{th}$  respectively. We choose a quadratic cost function and thus, an increasing marginal cost for the thermal production because: (i) the thermal production results from different fossil fuel generation technologies (e.g. coal, gas -combined cycle or not-, fuel oil), (ii) the high fixed costs of thermal production need to be recovered, (iii) we want to keep our model simple by choosing the simplest cost function for thermal (DGEMP & DIDEME (2003, 2008), MIT (2003, 2009), Cour des Comptes (2012)).

## 2.3 Notations and constraints

- $T$ : the time horizon of our model. Its length is chosen to be equal to 36 months<sup>9</sup> beginning by the month of January in order to obtain a sufficiently long time horizon to follow up the evolution of the value of the optimal solutions and at the same time to be consistent with the absence of the discount rate. The complexity of our model leads to compromise refinement of the model and computational capacity by choosing a reasoning in months<sup>10</sup> rather than weeks.
- $T_{campaign}$ : the time horizon of the campaign. A French nuclear producer has two main options regarding the scheduling of fuel reloading (Source: EDF (2008), CEA (2008)):
  - per third (1/3) of fuel reservoir (representing a reloading of reactor's core per third of its full capacity) that corresponds to 18 months of campaign and 396 days equivalent to full capacity for a unit of 1300 MW,
  - per quarter (1/4) of fuel reservoir (representing a reloading of reactor's core per quarter of its full capacity) that corresponds to 12 months of campaign and 258 days equivalent to full capacity for a unit of 1500 MW.

Both options<sup>11</sup> of fuel reloading result from the operational schema of EDF (Electricité de France) that is strategically chosen in order to optimize the allocation of the shutdowns of nuclear reactors for reloading (EDF (2008, 2010)). So, the scheduling of fuel reloading is entirely exogenous within our model (Bertel and Naudet (2004), CEA (2008)). Our goal is to determine the optimal allocation of the nuclear fuel stored in the reservoir during the

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<sup>9</sup>The time horizon of the model is a multiplicative of twelve, being expressed in months. Therefore it could be modified.

<sup>10</sup>This reasoning is also met in articles which study the optimal management of hydro-reservoirs in mixed hydro-thermal electricity systems (e.g. Arellano (2004), Bushnell (1998)).

<sup>11</sup>In the case of a unit of 900 MW, the scheduling of fuel reloading is the following: (i) 1/3 of fuel reservoir that corresponds to 18 months of campaign and 385 days equivalent to full capacity, (ii) 1/4 of fuel reservoir that corresponds to 12 months of campaign and 280 days equivalent to full capacity.

different campaigns of production for a reloading pattern provided by the French nuclear operator via the model ORION. We retain a duration of campaign equivalent to 12 months to get a cyclic model with a periodicity of one year. The one year period can be then decomposed into 11 months being the period of production and 1 month corresponding to the month of reloading of the fuel. We do not choose a campaign of 18 months because it is not in accordance with the “good” seasonal allocation of shutdowns of the nuclear units which consists of avoiding shutdowns in high demand periods (winter) and concentrating them as much as possible in low demand periods (between May and September). In fact, if the nuclear producer reloads fuel in summer when the demand is low the date of the next reloading will be then in winter when the demand is high. The case of having both a campaign of 12 and of 18 months is excluded in order to avoid complicate our model and because the choice of normative duration of the campaign can not be changed for a given nuclear reactor. The Nuclear Safety Authority (NSA)<sup>12</sup> has to give the authorization for any changes on the choice of duration of the campaign. Additional to that the optimal allocation of the shutdowns of all 58 nuclear reactors for reloading is decided in advance according to safety rules imposed by NSA.

- $D_t$ : the level of demand observed in month  $t = 1, \dots, T$ . The demand for electricity being an exogenous variable is assumed perfectly inelastic mainly because in the short-term to medium-term, we may consider that price variations can not be observed by consumers in real time and consumers habits and prior investments in electrical devices can not change immediately. If we include a price elasticity of demand in our model, it would have a random value since there are no particular elements that enable to assess its value.
- $Q_t^{hyd}$ : the hydro-production coming from the run-of-river<sup>13</sup> hydro plants in month  $t = 1, \dots, T$ . We assume that the monthly run-of-river hydro production is constant over the total time horizon of our model given: (i) the non-availability of the data with regard to the seasonal variations of hydro production because of precipitation and snow melting, (ii) its low volatility caused by a relatively low standard deviation which leads to a steady evolution of its monthly value near to the mean over a year. It is calculated by the mean of the yearly production. In this way, we deduce a significant part of the base load demand in order to have a more accurate picture of the demand served by the nuclear and thermal units. The intermittency that determines the base load production of the renewable energy plants makes our model more complex and additionally to this it is not coherent with the deterministic character of our model which is why we do not consider it.
- $q_{njt}^{nuc}$ : the level of the nuclear production during the month  $t = 1, \dots, T$  for the unit  $j$  of producer  $n$ .
- $Q_{max}^{n,j,nuc}$ : the maximum nuclear production that can be realized by the unit  $j$  of producer  $n$  during a month. The nuclear capacity is an exogenous variable.
- $Q_{min}^{n,j,nuc}$ : the minimum nuclear production that can be realized by the unit  $j$  of producer  $n$  during a month.
- $q_{nt}^{th}$ : the level of the thermal production during the month  $t = 1, \dots, T$  for the producer  $n$ .
- $Q_{max}^{n,th}$ : the maximum thermal production during a month for the producer  $n$ . It corresponds to the nominal thermal capacity of producer  $n$ . A producer may use the thermal resources

<sup>12</sup>The Nuclear Safety Authority (NSA) is tasked, on behalf of the state, with regulating nuclear safety in order to protect workers, the public and the environment in France.

<sup>13</sup>The run-of-river hydro plants have little or no capacity for energy storage, hence they can not co-ordinate the output of electricity generation to match consumer demand. Consequently, they serve as baseload power plants.



to produce electricity until it reaches the level of demand of the corresponding month respecting at the same time the constraint (2). The thermal capacity is an exogenous variable.

- $Q_{min}^{n,th}$ : the minimum thermal production during a month for the producer  $n$ . There is no minimum for thermal production  $Q_{min}^{n,th} = 0$ .

**The minimum and maximum production constraints have the form:**

$$\begin{cases} Q_{min}^{n,j,nuc} \leq q_{njt}^{nuc} \leq Q_{max}^{n,j,nuc}, & \text{if no reload during month } t \text{ for unit } j \\ q_{njt}^{nuc} = 0, & \text{if unit } j \text{ reloads during month } t \end{cases} \quad (1)$$

$$0 \leq q_{nt}^{th} \leq Q_{max}^{n,th} \quad (2)$$

- $S_{reload}^{n,j}$ : the nuclear fuel stock of reloading available to the unit  $j$  of producer  $n$ . This stock will be expressed thanks to the conversion between the quantity of energy and the corresponding number of days of operation at full capacity rather than expressing it in kilograms of uranium or number of nuclear fuel rods. In our model, the number of days of operation equivalent to full capacity is constant for all  $j, n$  and inferior than 11 months which permits and obliges at the same time to modulate the nuclear production. The nuclear fuel stock of reloading  $S_{reload}^{n,j}$  is equal to the corresponding capacity of the units of type  $j$  of producer  $n$  ( $\text{Capacity}^{n,j,nuc}$ ) multiplied by the number of hours equivalent to full capacity during a campaign. More precisely, one has:

$$S_{reload}^{n,j} = 1 \times \text{Capacity}^{n,j,nuc} \times \text{Number of days equivalent to full capacity} \times 24$$

which corresponds to the nuclear fuel stock of reloading over a campaign of production.

- $S_t^{n,j}$ : the quantity of fuel stored in the nuclear reservoir and available to the unit  $j$  of producer  $n$  at the beginning of the month  $t = 1, \dots, T$ . Evidently, we have  $S_t^{n,j} \geq 0$ . If  $t$  is the month during which the producer  $n$  reloads the fuel of the reactor then, the stock at the beginning of the following month (beginning of the campaign) is equal to  $S_{reload}^{n,j}$ . A producer has a quantity of nuclear fuel stock equal to zero at the end of a campaign (beginning of the month of reloading) which means that it spends all its nuclear fuel stock of reloading  $S_{reload}^{n,j}$  during the campaign. The reasons that lead us to this ascertainment mainly concern the implicit costs that result from not consuming the totality of the nuclear fuel stock during a campaign. Moreover, a producer has to finish the period  $T$  at least with the same quantity of nuclear fuel as the initial one ( $S_{T+1}^{n,j} \geq S_1^{n,j}$ ). The consideration of this constraint is motivated by some arguments analytically exposed in the second chapter (avoid to “over-consume” the nuclear fuel stock to reach the maximum nuclear production level because of induced negative effects, assure that each new cycle of simulations of 36 months starts with the same quantity of nuclear fuel ( $S_1^{n,j}$ )).

**The nuclear fuel constraints for the nuclear unit  $j$  of producer  $n$  are defined as follows:**

<b>j=1</b>	<b>j ∈ {2, ..., 11}</b>	<b>j=12</b>
$\sum_{t=2}^{12} q_{n1t}^{nuc} = S_{reload}^{n,1}$	$\sum_{t=1}^{j-1} q_{njt}^{nuc} = S_1^{n,j}$	$\sum_{t=1}^{11} q_{n12t}^{nuc} = S_{reload}^{n,12}$
$\sum_{t=14}^{24} q_{n1t}^{nuc} = S_{reload}^{n,1}$	$\sum_{t=j+1}^{j+12-1} q_{njt}^{nuc} = S_{reload}^{n,j}$	$\sum_{t=13}^{23} q_{n12t}^{nuc} = S_{reload}^{n,12}$
$\sum_{t=26}^T q_{n1t}^{nuc} = S_{reload}^{n,1}$	$\sum_{t=j+12+1}^{j+2 \cdot 12-1} q_{njt}^{nuc} = S_{reload}^{n,j}$	$\sum_{t=25}^{T-1} q_{n12t}^{nuc} = S_{reload}^{n,12}$
	$\sum_{t=j+2 \cdot 12+1}^T q_{njt}^{nuc} = S_{reload}^{n,j} - S_1^{n,j}$	

Table 1

We can see that the nuclear units of type  $\{2, \dots, 11\}$  have two additional constraints than the nuclear units of type 1 and 12. This is because there exist, at the beginning and end of the

game, campaigns that we will qualify as incomplete.

## 2.4 Number of optimization variables and of optimization constraints

In our model, the total number of optimization variables is equal to  $N \cdot (J \cdot T + T) = N \cdot (12 \cdot 36 + 36) = N \cdot 468$ . The number of constraints resulting from the equality between supply and demand is  $T = 36$ . In addition, the number of nuclear fuel constraints is  $N \cdot ((2 \cdot K + 1) \cdot (J - 2) + (2 \cdot K) \cdot 2) = N \cdot ((2 \cdot 3 + 1) \cdot (12 - 2) + (2 \cdot 3) \cdot 2) = N \cdot 82$ , where  $K$  represents the number of campaigns within our model. Lastly, the number of minimum and maximum nuclear and thermal production constraints is equal to  $N \cdot (J \cdot T + T) = N \cdot (12 \cdot 36 + 36) = N \cdot 468$ . Hence, the total number of optimization constraints is equal to  $N \cdot 550 + 36$ . Even in the case of a unique producer ( $N = 1$ ), the number of variables (468) and of optimization constraints (586) are quite large which leads to computational difficulties. This is because, the level of difficulty of the numerical program to compute a solution of an optimization problem is increasing with respect to the size of the model (number of optimization variables, number of optimization constraints).

In general, computational difficulties can result from: (i) the difficulty of the numerical program in calculating a global optimum since it can stop running when it finds a first solution which could be a local optimum of the optimization problem and not proceeding until it finds a global optimum, (ii) the sensibility of calculations with regard to the initial point that one chooses so that the program start running (different initial points can lead to different results), (iii) the duration of calculations which is increasing with respect to the size of the model.

## 3 Maximization of social welfare

In this section, we study the maximization of social welfare under production and nuclear fuel constraints as well as the supply-demand equilibrium constraints. Under the assumption that demand is inelastic, we show that the social welfare maximization problem is equivalent to the total cost minimization problem (same set of solutions) and therefore, we search for a solution of the total cost minimization problem. Then, we give a property that characterizes the optimal solutions when the production constraints are not binding what we call “interior” solutions. Specifically, we prove that in the absence of binding productions constraints, the solutions of the social welfare maximization problem (equivalently total cost minimization problem) are completely determined by a constant thermal production.

### 3.1 A property of the “interior” solutions

The maximization of social welfare is an optimization problem which consists in the maximization of the total surplus. Total surplus results from the sum of consumer surplus (denoted by SC) associated with a given level of production and the sum of producer surplus (denoted by SP). Consumer surplus is the difference between the total amount that consumers are willing and able to pay for electricity and the total amount that they actually do pay (electricity evaluated at the market price) (Renshaw (2005)). The surplus of producer is equal to its revenue minus the variable costs or equivalently to the profit increased by the fixed costs (Varian (2006), Renshaw (2005)). Without loss of generality, we may translate producer’s surplus by the fixed costs.

The social welfare maximization problem is

$$\max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( SC(\sum_{j=1}^J q_{jt}^{nuc} + q_t^{th}) + SP(\sum_{j=1}^J q_{jt}^{nuc}, q_t^{th}) \right)$$

We denote  $q_{jt}^{nuc} = \sum_{n=1}^N q_{njt}^{nuc}$  the aggregate nuclear production and  $q_t^{th} = \sum_{n=1}^N q_{nt}^{th}$  the aggregate thermal production obtained both during month  $t$ . For example, the exogenous variable  $Q_{max}^{th}$  will now represent the aggregate maximum thermal production. In the “aggregate” case which is a particular case, given the minimum/maximum nuclear production constraints ((1), (2)), the set of feasible solutions  $C$  of the social welfare maximization problem is defined as

$$C = \left\{ q \in M \text{ s.t. } \begin{array}{ll} Q_{min}^{j,nuc} \leq q_{jt}^{nuc} \leq Q_{max}^{j,nuc}, & \text{for all } j, t \\ 0 \leq q_t^{th} \leq Q_{max}^{th}, & \text{for all } t \end{array} \right\}$$

The set  $M$  is defined by all the production vectors of the form  $q = ((q_{j1}^{nuc})_{j=1}^J, \dots, (q_{jT}^{nuc})_{j=1}^J, q_1^{th}, \dots, q_T^{th})$  that respect the nuclear fuel constraints of table 1 as well as the supply-demand equilibrium constraint (3) for all  $t$

$$\sum_{j=1}^J \bar{q}_{jt}^{nuc} + \bar{q}_t^{th} = D_t - Q_t^{hyd}. \quad (3)$$

The set  $M$  is affine and the set  $C$  is compact (closed and bounded) and convex.

We also define  $F$  as the relative interior<sup>14</sup> of  $C$  ( $F = ri(C)$ ). It has the following form

$$F = \left\{ q \in M \text{ s.t. } \begin{array}{ll} Q_{min}^{j,nuc} < q_{jt}^{nuc} < Q_{max}^{j,nuc}, & \text{for all } j, t \\ 0 < q_t^{th} < Q_{max}^{th}, & \text{for all } t \end{array} \right\}$$

Note that if unit  $j$  reloads during month  $t$  then  $q_{jt}^{nuc} = 0$  and thus, the strict inequality constraints that determine the nuclear production  $q_{jt}^{nuc}$  in the set  $F$  are no more valid. Moreover, we remark that the non-emptiness of the set  $F$  obviously depends on the values of the exogenous variables  $(Q_{max}^{j,nuc}, Q_{min}^{j,nuc}, Q_{max}^{th}, S_{reload}^j, S_1^j, D_t, Q_t^{hyd})$  of the social welfare maximization problem. Theoretically, the set  $F$ , which is a subset of  $C$ , is non-empty. Following some linear transformations in the actual form of the optimization constraints included in the set  $F$  and then using the classical result of non-emptiness of the interior of unit simplex, we find a point contained in  $F$ . Empirically, in subsection 4, we show that  $F$  is a non-empty set for our numerical data, hence the above assumption complies with this particular data.

Consequently, we have to solve

$$\max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( \left[ \int_{p_t}^{\infty} D_t(p_t^*) dp_t^* \right] + \left[ p_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}) \right] \right)$$

where  $D_t(\cdot)$  is the demand function at time  $t$ . As we can see in the part of the objective function which corresponds to consumer surplus (the indefinite integral of the demand function from the price  $p_t$  to the reservation price), the reservation price of a consumer, i.e. the maximum price that a consumer is willing to pay for electricity, has an infinite value. This is because, in view

<sup>14</sup>It is important to emphasize that, in the general case of  $n$  producers, the usual interior of  $C$  is empty since  $M$  is an affine set that is not equal to  $\mathbb{R}^n$ . Consequently, we focus on a classical generalization called relative interior (for the notion of the relative interior of a set cf. for example Florenzano and Le Van (2001), Boyd and Vandenberghe (2004), Pugh (2002)).

of the assumption of inelastic demand, the demand function  $D_t(\cdot)$  is constant. We also recall that the price  $p_t$  is given by the equality between supply and demand during the time  $t$ .

However, the formula defining consumer surplus does not make sense in the presence of inelastic demand (infinite value of surplus). In view of this remark, we focus on the variation of consumer surplus. Nevertheless, the infinite value of consumers surplus leads to an indeterminate form of the variation of consumers surplus. In order to avoid this ambiguity, we define explicitly the variation of consumer surplus when the price evolves from  $\bar{p}_t$  which is a level of reference to  $p_t$  by the following formula ( $\Delta$ )

$$\Delta = - \int_{\bar{p}_t}^{p_t} D_t(p_t^*) dp_t^*$$

This definition is coherent with the classical case (finite value of surplus).

We can now start the calculation of the integral to obtain

$$\Delta = -D_t(p_t - \bar{p}_t) = \bar{p}_t D_t - p_t D_t = \bar{p}_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) - p_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) = K_t - p_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right)$$

where  $K_t = \bar{p}_t D_t = \bar{p}_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right)$ . It is obviously a constant that depends on  $t$ .

In view of these remarks, we will maximize the function

$$\sum_{t=1}^T \left( \left[ K_t - p_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) \right] + \left[ p_t \left( \sum_{j=1}^J q_{jt}^{nuc} + q_t^{th} \right) - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}) \right] \right)$$

Accordingly, the social welfare maximization problem can be written as

$$\max_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( K_t - \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) - C^{th}(q_t^{th}) \right) \quad (4)$$

or equivalently

$$\min_{((q_{jt}^{nuc})_{j=1}^J, q_t^{th})_{t=1}^T \in C} \sum_{t=1}^T \left( \sum_{j=1}^J C_j^{nuc}(q_{jt}^{nuc}) + C^{th}(q_t^{th}) \right) \quad (5)$$

Therefore, we deduce that the social welfare maximization problem is equivalent to the total cost minimization problem (5) (same set of solutions). If the solution of the social welfare maximization problem belongs to the set  $F$  in which the production constraints are not binding, we obtain a property given by the following proposition.

**Proposition 3.1** *If there exists a solution  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T \in F$  such that the social welfare is maximum on  $C$  then  $\hat{q}_1^{th} = \hat{q}_2^{th} = \dots = \hat{q}_T^{th}$ .*

### Proof

A proof of this proposition is provided in the Ph.D. thesis on page 207 – 210 (Lykidi (2014)).  $\square$

We remind that  $C$  is a compact set, thus the total cost minimization problem has solutions on  $C$ . Nevertheless, it may not have solutions on the set  $F$  since it is not compact. Hence, the existence of a solution of the problem (5) on  $F$  has the form of assumption in Proposition 3.1.

### 3.1.1 Economic interpretation of the Lagrange multipliers of the social welfare maximization problem (or the equivalent total cost minimization problem (5))

In view of the proof of Proposition 3.1, we can interpret economically the Lagrange multipliers of the equivalent to the social welfare maximization problem, total cost minimization problem (5). We remind that  $\bar{\mu}_t$  is the Lagrange multiplier associated with the supply-demand equilibrium constraint at each month  $t$  and  $\bar{\lambda}_j^k$  is the Lagrange multiplier for the nuclear fuel constraint of the unit  $j$  during the campaign  $k$ . Since  $\hat{q} \in F$ , the equation (3.20) (respectively (3.21)) observed on page 209 of the proof implies that the sign of the multiplier  $\bar{\mu}_1$  (respectively  $\bar{\mu}_2$ ) is strictly positive. By a symmetric argument, the Lagrange multiplier  $\bar{\mu}_t$  is strictly positive ( $\bar{\mu}_t > 0$ ) for all  $t$ . Hence, in view of equations (3.22) and (3.23) on page 209 of the proof, the multiplier  $\bar{\lambda}_3^1$  (respectively  $\bar{\lambda}_j^k$ ) is strictly negative. Indeed, if an additional unit of nuclear fuel became available for unit  $j$  during campaign  $k$ , the thermal production would decrease which would lead to the augmentation of the nuclear production cost and the diminution of the thermal production cost. However, the second effect that regards the decrease of the thermal production cost is the most important. Consequently, the “additional” cost resulting from an additional nuclear fuel unit and thus the value of the multiplier  $\bar{\lambda}_j^k$  should be negative. The multiplier  $\bar{\lambda}_j^k$  indicates the “marginal value of nuclear fuel stock”, i.e. the additional cost  $|\bar{\lambda}_j^k|$  unit  $j$  would incur if the nuclear fuel stock decreased by one unit during the campaign  $k$ .

Let us now proceed with a proposition which shows that a constant thermal production is a sufficient condition for optimality on  $C$ .

**Proposition 3.2** *If  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  is a production vector belonging to  $C$  such that  $\hat{q}_1^{th} = \hat{q}_2^{th} = \dots = \hat{q}_T^{th}$  then  $((\hat{q}_{jt}^{nuc})_{j=1}^J, \hat{q}_t^{th})_{t=1}^T$  is a solution of the social welfare maximization problem (equivalently total cost minimization problem) on  $C$ .*

#### Proof

A proof of this proposition is provided in the Ph.D. thesis on page 211 – 213 (Lykidi (2014)).  $\square$

**Remark 3.1** *We can prove that the strict convexity of the total production cost function with respect to the thermal production  $q^{th}$  implies the unicity of solutions with respect to the thermal component (Lykidi (2014)). Nevertheless, if we take into account the other variables which do not influence the total production cost, the total production cost function is convex with regard to  $q$ , which does not mean necessarily that the entire solution is unique.*

### 3.1.2 Economic analysis of Proposition 3.1 and of Proposition 3.2

In view of Propositions 3.1 and 3.2 (pages 10, 11), we conclude that in the absence of binding production constraints, a constant thermal production is a characteristic property of solutions of the social welfare maximization problem. On the contrary, the nuclear production, which is the first that is called to satisfy demand (according to the merit-order rule), is adjusted fully to the seasonal variations of demand. This is a result of the behaviour of producers who use the thermal capacity to produce the same quantity every month in order to meet demand. Note that it means that the amplitude of demand has to be smaller than the nuclear capacity so

that the equality between supply and demand is respected each month (more precisely, the amplitude of demand has to be inferior than the amplitude of nuclear production that can be realized  $[\max((\sum_{j=1}^J Q_{max}^{j,nuc}(t))_{t=1}^T) - \min((\sum_{j=1}^J Q_{min}^{j,nuc}(t))_{t=1}^T)]$  given our numerical modelling. Furthermore, this property signifies that thermal is the marginal technology even during seasons of low demand. Consequently, prices are determined permanently by the marginal cost of fossil fuel technologies and hence, they stay constant during the entire time horizon of the model.

## 4 Numerical modelling

In this section, we study the nuclear and thermal production levels as well as the storage levels resulting from the social welfare maximization problem, within a simple numerical model solved.

### 4.1 Data

The data that we use within our numerical model is derived from the French electricity market and it is collected by different entities and for different years because of the difficulty of collection. Specifically, consumption data is given by the French Transmission & System Operator (named RTE) for the year 2007, the annual generation capacity of hydro (run-of-river) results from the French nuclear operator (EDF) while the annual nuclear and thermal (coal and gas) generation capacity comes from RTE for the year 2009, the nuclear fuel stock of reloading has been provided by EDF for the same year. The fixed and variable costs of nuclear, coal and gas generation are obtained by the official report “Reference Costs of Electricity Production” issued by the ministry of industry (General Direction for Energy and Raw Materials (DGEMP) & Directorate for Demand and Energy Markets (DIDEME)) and they are computed for the year 2007. In view of the specific characteristics of the nuclear generation technology and of its production cost, we also provide a short analysis regarding the impact of the discount rate on the calculation of the nuclear cost, the economic consequences of a load-following mode of operation of nuclear reactors on the nuclear cost as well as the main points of differentiation between nuclear and thermal production cost (Bertel and Naudet (2004)) in the Ph.D. thesis of Lykidi (2014). Then, we present in detail some specific data assumptions considered for our numerical modelling with respect to: (i) the value of the exchange rate, of the discount rate, of the cost of CO<sub>2</sub> per ton and the price of coal and gas (ii) the computation of the coefficients of the thermal production cost, (iii) the simulation of the capacity for each type of nuclear unit and of the initial value of the nuclear fuel stock ( $S_1^j$ ), (iv) the calculation of the number of days equivalent to full capacity, (v) the technical minimum and maximum for an EPR reactor in order to determine the minimum and maximum nuclear production constraints. Finally, we refer to a couple of economical results which can be concluded within our data base and which we fully develop in the second chapter of the Ph.D. thesis of Lykidi: (i) the calculation of the average nuclear cost here (37.25 euros per MWh) is near the scope of nuclear electricity prices (37.5 - 38.8 euros per MWh) evaluated for the NOME<sup>15</sup> law (Commission for Energy Regulation (CRE) evaluate this range of prices in 2010 (before Fukushima accident in 2011 (Les Echos (20/04/2011)) in order to recommend to EDF a just price for selling nuclear capacity

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<sup>15</sup>The “Nouvelle Organisation du Marché de l’Electricité” (NOME) law indicates the findings of the report of the Commission Champsaur which suggests access to nuclear electricity of the French nuclear operator (EDF) for all producers (Champsaur (2009)). Specifically, the NOME law forces EDF to sell at a competitive price to alternative producers of electricity and gas (GDF Suez, E.ON, ENEL, Poweo, Direct Energy, etc.) a quarter of its nuclear production until 2025. This price should include the total cost of the operating nuclear plants.

to other producers (Le Monde (01/02/2011))), (ii) the total monthly thermal production cost can be covered only in the case that alternative generation technologies with higher marginal costs are called to satisfy demand (e.g. hydro-storage units, oil, etc.).

## 4.2 Simulation results

In our numerical model, we maximize social welfare (equivalently we minimize total production cost) on the entire set of feasible solutions  $C$ .

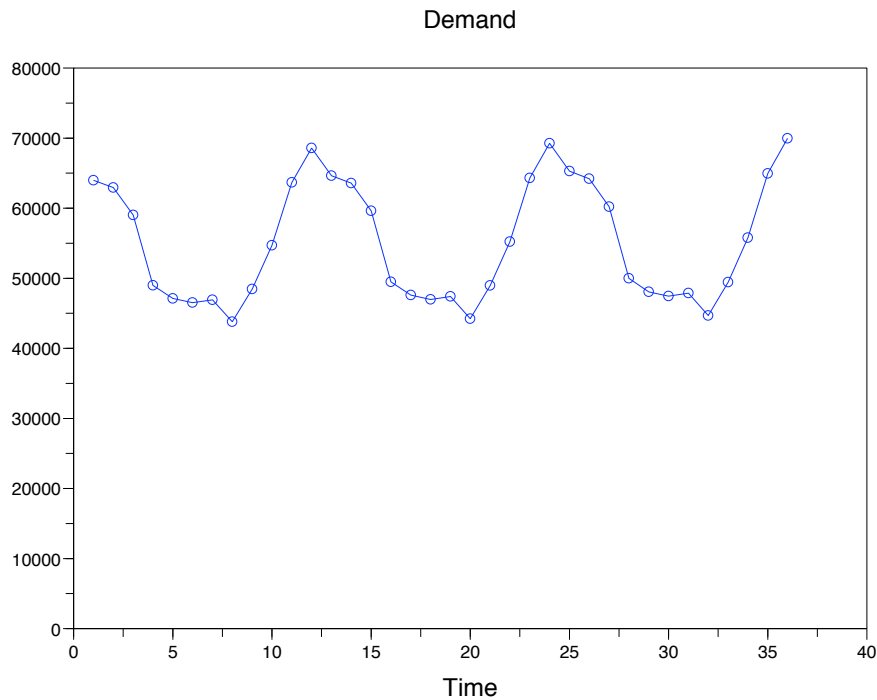


Figure 1: Simulated demand (in MW)

Simulation results obtained by our numerical model show that nuclear follows entirely the seasonal variations of demand<sup>16</sup> by decreasing during summer and increasing during winter while thermal has a constant level of production during the entire time horizon of our model. The amount of nuclear capacity is such that the amplitude of demand does never exceed it within our numerical model which means that in view of the constant level of thermal production, the equality between supply and demand is always respected with only nuclear operated in load-following mode. Thus, thermal remains the marginal technology during the entire time horizon of the model while the nuclear technology is never marginal, even during months of low demand (see Figure 1, Figure 2, Figure 3, Figure 4). We also observe that both nuclear

<sup>16</sup>The amounts of monthly demand  $D_t$  are obtained for the period January 2007 - December 2009. In particular, the values of monthly demand during the period January 2007 - December 2007 come from our historical data. Then, we reproduce these values by applying a positive rate of 1% per year on the monthly demand for the years that follow (2008 and 2009) to take into account the increasing trend of demand from one year to another. We did a rescaling on this data to take into account the diversity on the length of the months. Note also that the monthly demand in 2007 results from the aggregation of the hourly demand found within our historical data.

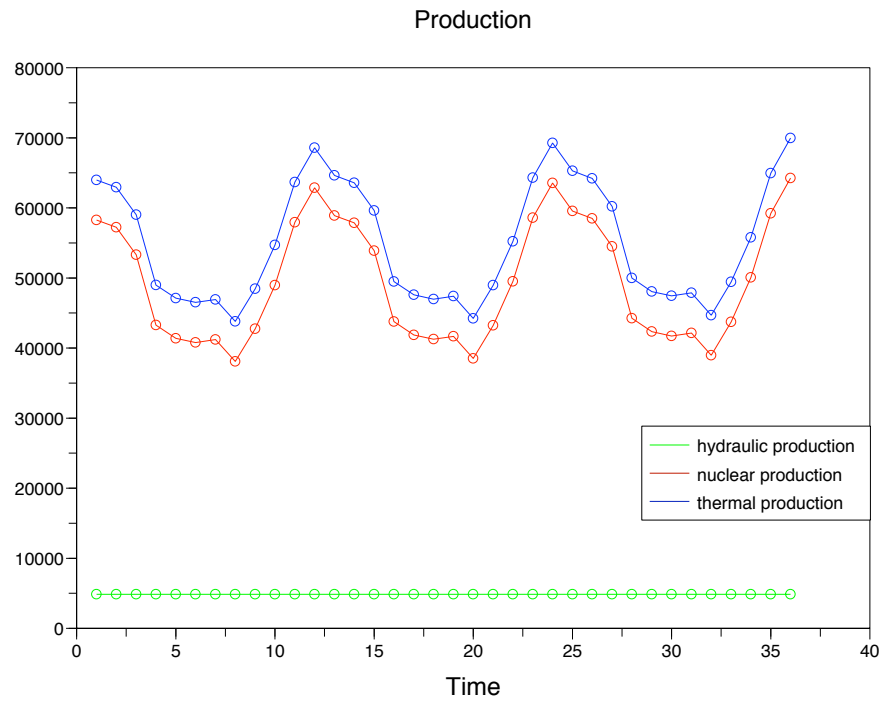


Figure 2: Simulated hydro(run-of-river)/nuclear/thermal production (in MW)

and thermal production do not saturate the minimum/maximum nuclear<sup>17</sup> and thermal<sup>18</sup> pro-

<sup>17</sup>The maximum nuclear production during the month  $t$  given that some unit is inactive during this month

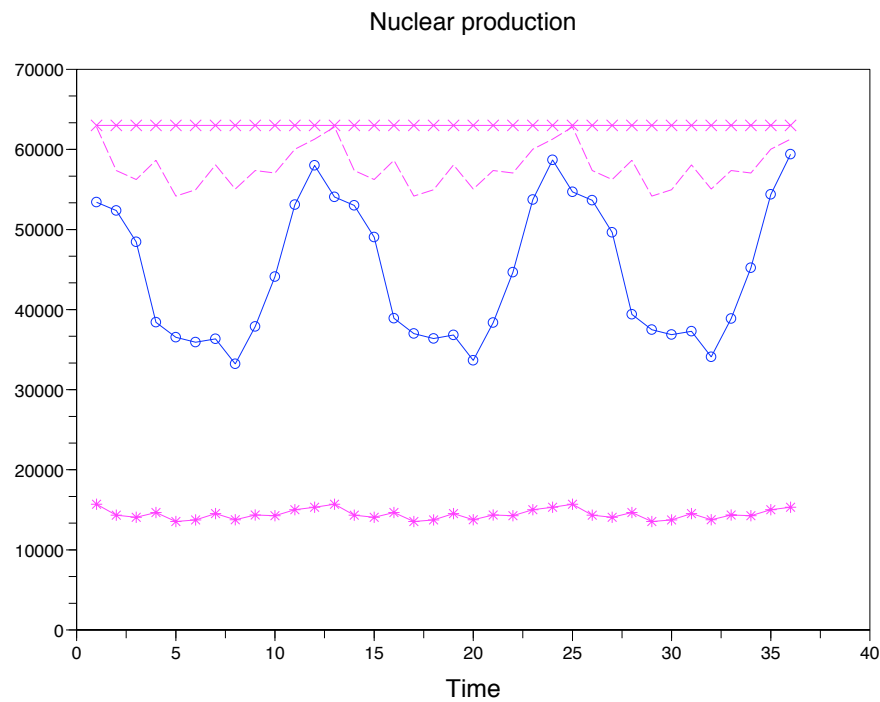


Figure 3: Simulated nuclear production (in MW)



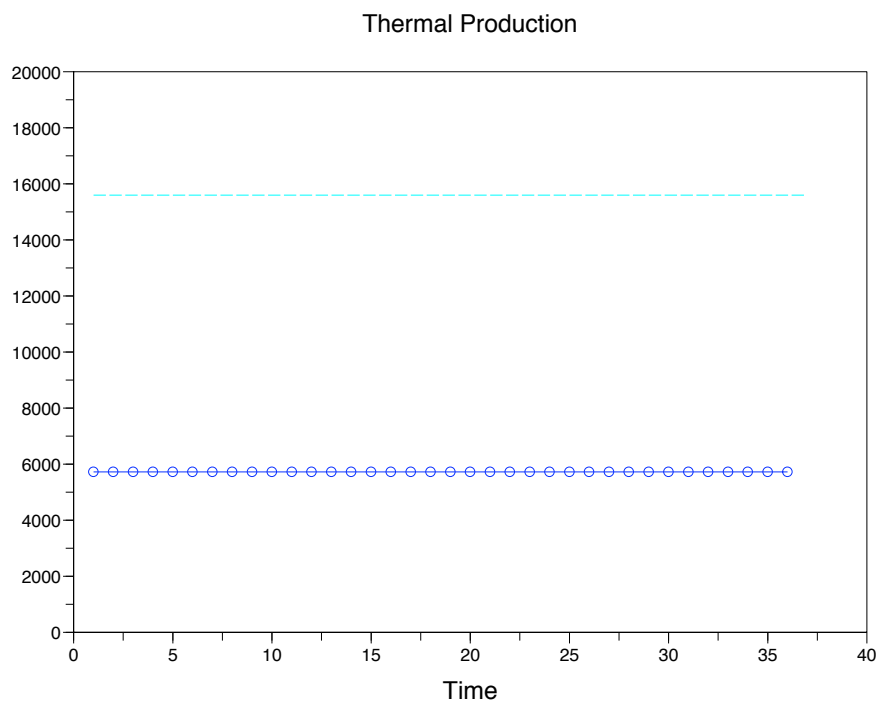


Figure 4: Simulated thermal production (in MW)

(month of reloading) is represented by the purple dotted line. This quantity is obviously below the nominal capacity of the French nuclear set represented by the crossed purple line. The minimum nuclear production

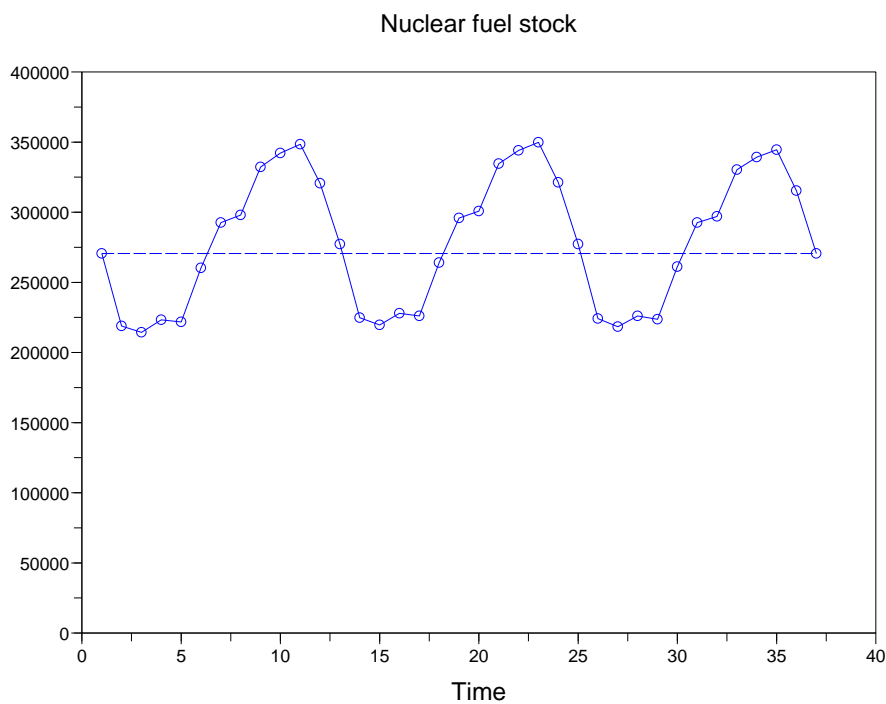


Figure 5: Simulated nuclear fuel stock (in MW)

duction constraints. Consequently, we verify (through a numerical test too) that the numerical solution described in this section belongs to  $F$ , therefore our simulation results are in accordance with Proposition 3.1 presented on page 10 seeing that the thermal component of this solution is constant. We also deduce that our numerical results are in line with Proposition 3.2 appeared on page 11 since a production vector which belongs to  $C$  and is characterized by a constant thermal production constitutes a solution of the social welfare maximization problem within our numerical model.

The essentially periodic evolution of the nuclear production results in “high” levels of nuclear fuel stock during summer and “low” levels of nuclear fuel stock during winter because of the seasonality that characterizes the variations of the nuclear production (high production during winter – low production during summer). Consequently, we observe a periodic evolution for the nuclear fuel stock as well as an oscillation around the “stock of reference”<sup>19</sup> (see Figure 3, Figure 5).

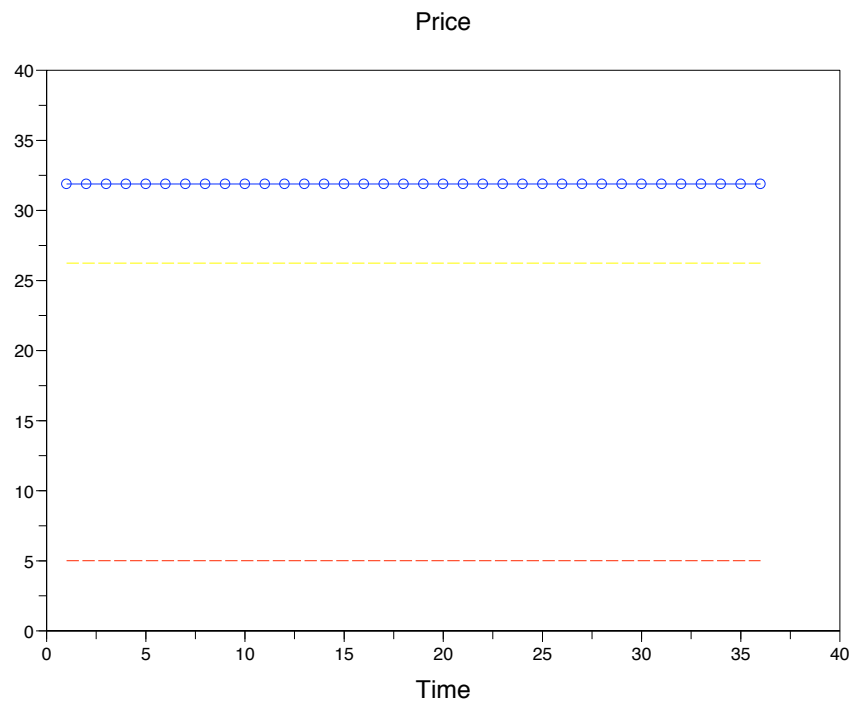


Figure 6: Simulated price (in Euro/MWh)

In view of the merit order price rule, the price<sup>20</sup> is determined by the thermal marginal cost since thermal is permanently the marginal technology (see Figure 6). Additionally to this, we can see that the price is constant during the entire period  $T$  because thermal is characterized by a constant level of production which leads to a constant thermal marginal cost.

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during the month  $t$  given that some unit is inactive during this month (month of reloading) is represented by the purple line of asterisks.

<sup>18</sup>The maximum thermal production during a month is represented by the white blue dotted line and corresponds to the nominal thermal capacity (including coal, gas, fuel, etc.) of the French set.

<sup>19</sup> The “stock of reference” is represented by the blue dotted line which shows the value of stock at the beginning, being also the value of stock at the end.

<sup>20</sup>The red (respectively yellow) dotted line indicates the price level when nuclear (respectively thermal) is the marginal technology.

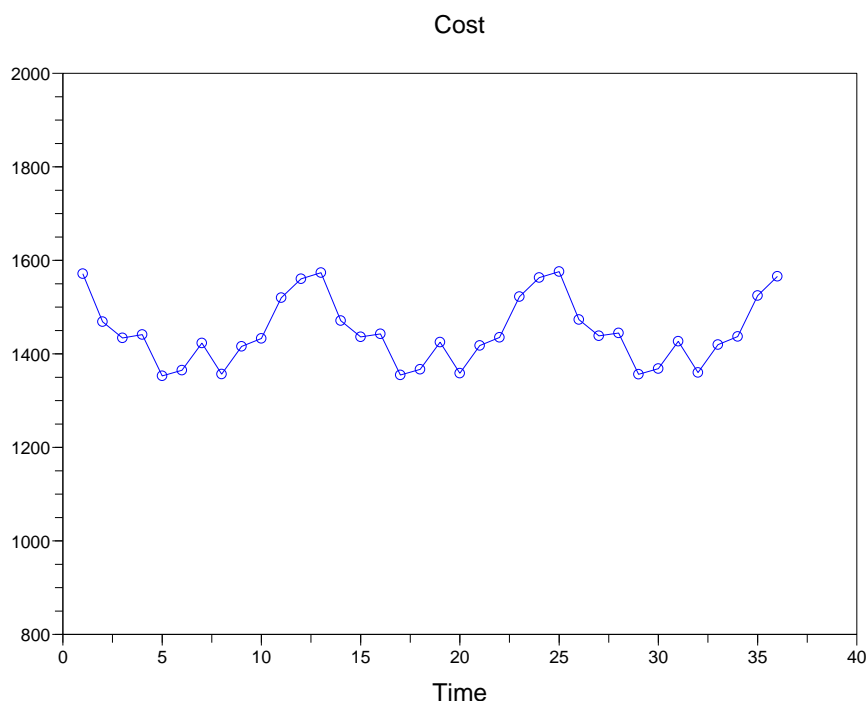


Figure 7: Total cost (in Euro (million))

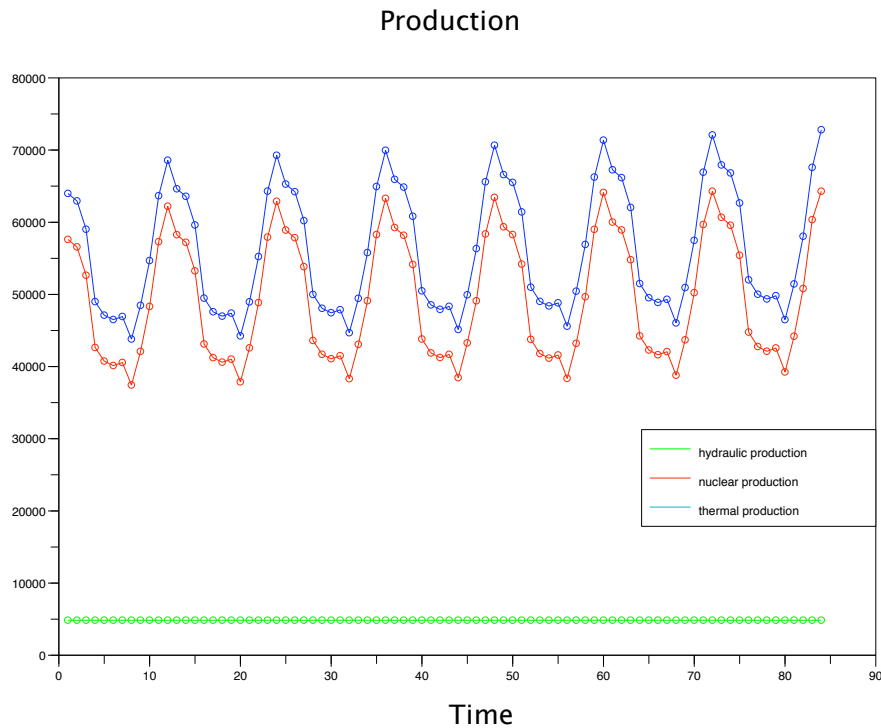
The evolution of total cost is almost periodic and its value increases during winter when we observe high levels of nuclear production and decreases during summer when we notice low levels of nuclear production (see Figure 7). As expected, the values of total cost (respectively total variable cost) resulting from the optimal short-term production problem and the optimal inter-temporal production problem are higher than the optimal value of total cost (respectively total variable cost) determined in this section (see Table 2). Equivalently, we can say that the values of social welfare obtained in the optimal short-term production problem and the optimal inter-temporal production problem are lower than the optimal value of social welfare given by the resolution of the social welfare maximization problem. To end, we notice that the total cost (respectively total variable cost) resulting from the optimal inter-temporal production problem is relatively higher than the total cost (respectively total variable cost) coming from the optimal short-term production problem. This implies that the value of social welfare is relatively lower when we maximize the inter-temporal profit where we determine a global optimum of the optimal production problem than when we maximize the current monthly profit where we look at “local” solutions of the optimal production problem.

	Social welfare maximization problem	Optimal inter-temporal production problem	Optimal short-term production problem
Total cost (in Euro)	$5.209 \times 10^{10}$	$5.261 \times 10^{10}$	$5.250 \times 10^{10}$
Total variable cost (in Euro)	$1.023 \times 10^{10}$	$1.075 \times 10^{10}$	$1.064 \times 10^{10}$
Total fixed cost (in Euro)	$4.186 \times 10^{10}$	$4.186 \times 10^{10}$	$4.186 \times 10^{10}$

Table 2

#### 4.2.1 General remarks

In view of the remark 3.1, the solution is unique with respect to the thermal component but considering the other variables which do not act on the total cost the whole solution is not definitively unique.

Figure 8: Simulated hydro(run-of-river)/nuclear/thermal production (in MW) ( $T=84$ )

Let also us notice that despite the possible modifications of the length of the time horizon  $T$  of the model, we do not obtain different production patterns. Therefore, we do not observe any changes in the periodic evolution of the nuclear and thermal production in the case of a

longer time horizon (e.g. for  $T = 84$ , see Figure 8).

### 4.3 Social welfare maximization problem VS Optimal inter-temporal production problem

In this section, we compare the theoretical and numerical results coming from the resolution of the optimal inter-temporal production problem and the social welfare maximization problem. In the first optimization problem, we determine the production levels that maximize the inter-temporal profit of a producer under production and storage constraints (Lykidi (2014)). Specific interest is given to constraints inherent to public interest and social welfare and in particular those imposed by the equality between supply and demand at each month even in a competitive market. These constraints are due to the important size of the french nuclear set which ensures the majority of the domestic demand. The second optimization problem however, gives priority to production decisions that fully maximize social welfare -and not only producers profit- under identical constraints. Here, we take into account that the production decisions of a very large nuclear set have a very significant effect on the equilibrium of the whole national electricity system and consequently on the welfare of the society.

From a theoretical point of view, we find that the optimal production behaviour deduced from the inter-temporal profit maximization problem is diametrically opposite to the optimal production behaviour resulting from the social welfare maximization problem. More precisely, when the minimum and maximum production constraints are not saturated, we observe that the solutions of the optimal inter-temporal production problem are fully characterized by a constant nuclear production while the solutions of the social welfare maximization problem are entirely characterized by a constant thermal production. So, producers maximize their inter-temporal profit by using nuclear as a baseload generation technology that produces always at a steady rate leaving thermal to follow-up the seasonal variations of the demand. On the contrary, social welfare is maximized through a totally flexible management of nuclear production which is adapted completely to demand's seasonal variations resulting from a constant thermal production that covers the residual demand every month. Nevertheless, in both problems, thermal is always the marginal technology, i.e. the generation technology that determines the market price over the entire time horizon of the model.

Numerically, the comparison of the optimal production behaviour between the inter-temporal profit maximization problem and the social welfare maximization problem does not provide exactly the same conclusions since the optimal inter-temporal production behaviour is not identical with the one resulted from the theoretical resolution of the problem. In our numerical model, the optimal production deduced from the “regularized” optimal inter-temporal production problem is such that both nuclear and thermal production follow the variations of demand in the medium-term. Optimal solutions of the problem that are characterized by a constant nuclear production do not exist. On the contrary, the optimal solutions derived from the numerical resolution of the social welfare maximization problem are not differentiated from those coming from the theoretical resolution of this problem since in both cases thermal remains constant during the whole period  $T$  and hence nuclear follows entirely the seasonal variations of demand. Consequently, in both numerical problems, nuclear units realize a load-following operation. Thermal units adapt their production output to follow-up load in the inter-temporal profit maximization problem while they produce the same quantity to meet demand every month in the social welfare maximization problem. Regarding the marginality duration of nuclear, we notice that it is significantly increased in the optimal inter-temporal production problem. More precisely, nuclear is the marginal technology during periods of low

demand, i.e. during the half of our model's period  $T$ , while the maximization of social welfare shows that thermal is the marginal technology during the total time horizon of our model.

At this point, it should be noted the paradoxical nature of both theoretical solutions which consists of not modulating production (thermal and nuclear respectively) over the whole time horizon  $T$ . It shows clearly the importance of the choice of capacity (missing here because capacities are exogenous variables within our model) in order to avoid to build and maintain unused capacity taking into consideration the fixed costs of generation technologies. From a theoretical point of view, in the social welfare maximization problem, if the level of nuclear capacity is sufficiently high, we obtain a constant thermal production since nuclear production can adjust fully to the seasonal variations of demand without exceeding its minimum and maximum production levels. In our numerical model, a decrease of 47% in nuclear capacity (compensated by the thermal capacity) would impose to modulate the thermal production in order to balance supply and demand and respect at the same time the production constraints. Similarly, from a theoretical perspective, in the optimal inter-temporal production problem, under the assumption that the amount of thermal capacity is sufficiently important, the thermal units load-follow without violating the minimum and maximum production constraints, thus leaving the nuclear units to produce at a constant rate. However, lower levels of thermal capacity would result in a modulation of nuclear production so that the supply-demand equilibrium constraints and the production constraints are satisfied. Indeed, in our numerical example, we remark that the level of thermal capacity, being lower than the amplitude of demand, results in a modulation of both nuclear and thermal production in order to satisfy the equality between supply and demand.

## 5 Conclusion

In a time period during which nuclear as an electricity generation technology is questioned significantly given the accident of Fukushima, we decided to determine the optimal production levels that maximize social welfare in a country where the global equilibrium between supply and demand depends totally on nuclear generation. The maximization of social "benefits" and the maximization of producers own profits under the same constraints consisted two different approaches that the social planner may look at and they led to very different optimal production behaviours.

Initially, we showed that the problem of maximization of social welfare can not be resolved within a framework of inelastic demand. To overcome this difficulty issued from this basic assumption of our model, we focused on the resolution of the total cost minimization problem by proving that it is equivalent to the problem of social welfare maximization (same set of solutions).

On a theoretical level, we resolved the social welfare maximization problem in the absence of binding production constraints which is a specific case (implying that capacities are significant). In this case, we proved that the optimal solutions are fully characterized by a constant thermal production. Therefore, under the assumption that the nuclear capacity is significant, the nuclear production follows fully the variations of demand permitting to the thermal production to be constant. Obviously, the optimal production scheduling determined by the social welfare maximization problem and the optimal inter-temporal production problem are completely opposite. Indeed, under the assumption that the production constraints are not saturated, social welfare (respectively inter-temporal profit) is maximized when the thermal (respectively nuclear) units produce at a constant rate. Therefore, in the first problem, given that the nuclear

capacity is sufficiently important, we have a totally flexible operation of nuclear plants and in the second problem, if the thermal capacity is sufficiently high, thermal production can adapt totally to demand's variations so that customers requirements for electricity are always satisfied in both cases. Clearly, thermal is the marginal technology during the entire time horizon  $T$  in both optimal production problems.

From an empirical point of view, the thermal units realize the same amount of production each month to cover the residual levels of demand, forcing nuclear to entirely follow the seasonal variations of demand. Here, the amplitude of demand is not greater than the nuclear capacity and hence, given the constant level of thermal production, the load-following operation of nuclear units does not lead to a violation of the minimum and maximum nuclear production constraints in order to equilibrate supply and demand. In our numerical example, a significant decrease of 47% of the nuclear capacity (compensated by the thermal capacity) would necessitate the simultaneous modulation of the thermal production in order to balance supply and demand every month. This has shown the robustness of our results since they would change only if an important reduction of nuclear capacities occurred.

The numerical resolution of the social welfare maximization problem showed a production behaviour that is identical with the one resulting from its theoretical resolution. The fact that nuclear capacities are very important with respect to thermal capacities in the French electricity market leads to a "paradoxical" optimal production behaviour being that of a steady thermal production which induces an entirely flexible nuclear production. For the same reason, in the case of the inter-temporal profit maximization problem, we did not arrive numerically at the same conclusions with those derived from its theoretical resolution. In France, the thermal capacity is not enough in order to manage nuclear uniquely as a baseload generation technology at the optimum. It has also to operate at semi-base load following a part of the variable demand, therefore, the flexible operation of nuclear units is inevitable. Hence, numerically, nuclear plants operate to follow load in both cases while thermal plants produce at a constant rate in the first case (social welfare maximization problem) while they follow load in the second case (inter-temporal profit maximization problem). The nuclear production is never marginal when maximizing social welfare and it is marginal during low demand periods when maximizing inter-temporal profit.

Our theoretical and numerical results showed that social optimum is ensured by investing significantly in nuclear capacity which shows clearly the necessity of nuclear in the energy mix of a country from the social welfare perspective within our model. On the contrary, in a decentralized market, the optimum of producers is attained if we make significant investments in thermal capacity (French case). Thus, we conclude that the amount of investments used to add new capacities of a generation technology (e.g. nuclear) plays an important role in the determination of the optimal production behaviour of the corresponding technology and of the other generation technologies (e.g. thermal) that constitute the national energy mix.

Nonetheless, we do not really see such behaviours in the French electricity market. For example, the report of CRE in 2007 indicates a duration of marginality of nuclear equivalent to a total of 1 or 2 months per year. This time period is not so close to the duration of nuclear marginality resulting from the theoretical and numerical results of the optimal inter-temporal production problem (a total duration of 6 months per year). Moreover, the thermal units do not always produce at a constant rate as we derived from the production behaviour that maximizes social welfare. The fact that nuclear producers and particularly the French nuclear operator (EDF) does not take into consideration assumptions and factors of our model which limit the model to a certain degree but contribute to resolve the above optimization problems and find an optimal solution is the main reason why we notice these divergences from the real world.

From an economical point of view, all these different approaches suggested in our model and the results which are deduced theoretically and numerically give insights in order to induce conclusions for policy and industry and thus, it could be interesting for the system operator to look at.

To conclude, in all cases, nuclear fuel modelled as a “reservoir” of energy follows the seasonal variations of demand in a competitive electricity market where nuclear capacity exceeds thermal capacity to a significant degree. But even if nuclear power does not possess the greatest part of the energy mix of a country (like France), it can be still operated at semi-base load following a part of demand’s variations because, technically, modern nuclear reactors are capable of flexible operation. This could lead to a more significant use of nuclear in the electricity production of a country and therefore a higher share of nuclear power as a percentage of its national energy production especially since nuclear promotes: (i) reduction of CO<sub>2</sub> emissions, (ii) energy independence from fossil fuel generation technologies, (iii) large-scale deployment of intermittent electricity sources (renewable energy), (iv) economic competitiveness of a country’s energy sector. All these factors play a very important role in the future of nuclear energy worldwide.

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